

ON THE ANOMALOUS ABSORPTION OF GAMMA-PHOTONS

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ABSTRACT. The absorption co-efficient of RaC gamma-rays penetrating through more than 16 cm. and up to 20 cm. of lead was experimentally measured with Geiger-muller counter. The counter was placed in different distances from the source and in vertical and horizontal positions in order to investigate the nature of secondary radiations. The results obtained show that the heterogeneity of the photon beam and the secondary radiation combined together has two opposite effects on the apparent absorption co-efficient obtained by gradually increasing the thickness of absorber. One predominates over the other depending on the position of the counter. When the counter is nearer to the absorber surface the apparent absorption co-efficient increases with increasing thickness of Pb and when further away it decreases with increasing thickness. These investigations may, therefore, have some significance in Rossi transition curve for cosmic-rays. When the counter is at a distance 45 cm. away from the source both for horizontal and vertical positions of the counter the absorption co-efficient of gamma-rays filtered through 19 cm. Pb is $416 \pm .028 \text{ cm}^{-1}$ which is more than 10% less than the theoretical minimum absorption co-efficient for gamma-rays in Pb. It is concluded that so much difference may be mainly due to positron annihilation radiation. Incidentally it is pointed out that since a slow positron gains in energy by capturing an electron the subsequent annihilation quanta may be partly responsible for the backward radiations in cosmic-rays and for the excess of low energy Compton electron associated with a cascade.

Another interesting fact noticed in the last experiment is that with the source in proper position the background rate of counting which remains nearly steady after 24.8 cm. of Pb is about double to that due to cosmic-rays alone. This may be merely due to some multiple scattered quanta but the possibility of these residual radiations being partly meson-like or neutron-like is not excluded.

INTRODUCTION

Since the discovery of gamma-rays, its absorption in different materials had been investigated by many workers and a nice summary of all the earlier works is given by Rutherford, Chadwick and Ellis (1932). In all these experiments, electroscopes and ionisation-chambers of comparatively large dimensions were used for the measurement of intensity and the absorption of gamma-rays from Ra (B+C) and ThC in lead, mercury, aluminium, etc., was studied under various experimental devices to eliminate the effect of so-called degraded radiations of secondary origin. These degraded radiations are secondary gamma-rays of longer wave length produced in the material

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when a beam of gamma-rays passes through it and their effect is to increase the absorption co-efficient μ defined by the well-known exponential relation $I = I_0 e^{-\mu d}$. But the numerical values for the absorption co-efficient obtained by various workers were rather conflicting and on analysis it was found that the absorption co-efficient depends mainly on two factors, *e.g.*, (1) degree of filtration and (2) the geometry of experimental arrangements. Factor (1) is due to the fact that we can never isolate strictly mono-chromatic gamma-ray source and the softer component of heterogeneous beam generally used is gradually cut out as the absorber thickness increases. As for example if we consider RaC gamma-rays, with which most of the experiment has been done, then, according to Ellis and Aston as mentioned by Rutherford, Chadwick and Ellis (*loc. cit.*) only 7.3% of the emitted gamma-rays are of energy 2.22 Mev and the rest are distributed over widely different energies. Skobelzyn (1927) obtained the evidence of a very feeble intensity of gamma-rays of energy 3 Mev. According to more recent investigation by Alichanow, Latyshev and others (1917) the hardest fraction of RaC gamma-rays is of energy 2.42 Mev. Similarly their investigations also showed that ThC gamma-rays of which the bulk of radiations are of energy 2.65 Mev, also contain a fraction roughly 25% of energy about 1.5 Mev to 2.2 Mev. Nowadays comparatively homogeneous gamma-rays are also available from the artificially radioactive element like sodium, etc., but even a homogeneous beam in passing through the matter becomes heterogeneous. In cosmic-rays also when a large burst or shower is generated by an energetic particle, we get a heterogeneous beam of photons partly due to Bremsstrahlung and partly due to positron annihilation radiations. Of course the intensity is comparatively small. The dependence of absorption co-efficient on the geometry of experimental arrangement is due to the mechanism of gamma-ray absorption as will be clear from the following brief discussion on the theory of gamma-ray absorption.

In recent years the inter-action of photons with matter has been thoroughly investigated from theoretical point of view and the results obtained have been very fruitful in explaining interesting cosmic-ray phenomena. In the region of gamma-photons it is now well-known that the absorption consists of three different processes namely, (1) photoelectric process, (2) Compton scattering and (3) pair formation and in higher energy region the latter two processes alone are important. The probability of Compton scattering was calculated by Compton, Dirac and finally by Klein and Nishina (1927) according to relativistic quantum mechanics. The probability of a photon of energy K_0 being scattered at an angle θ as a degraded quanta of energy K is given by the formulae.

$$d\phi = \frac{\gamma_0^2}{2} d\Omega \frac{K^2}{K_0^2} \left(\frac{K_0}{K} + \frac{K}{K_0} - \sin^2 \theta \right) \quad \dots (1)$$

$$K = \frac{K_0 \mu}{\mu + K_0(1 - \cos\theta)}$$

μ is the rest energy of an electron and $d\Omega$ the element of solid angle. Now if the cross-section (1) is integrated over all angles the total probability of a photon being lost by scattering per electron is obtained and in the higher energy region it is of the form

$$\phi = \phi_0 \frac{3}{8} \frac{\mu}{K_0} \left(\log \frac{2K_0}{\mu} + \frac{1}{2} \right) \quad \dots (2)$$

which shows that the probability of Compton scattering decreases as the energy of the quantum increases. On the other hand the probability of pair formation is proportional to the square of atomic number and rapidly increases as the energy of the photon increases. Therefore due to these two opposite effects gamma-photons have a minimum absorption co-efficient and as calculated by Heitler (1944), the minimum absorption co-efficient in lead is $.475 \text{ cm}^{-1}$ for energy about 3 Mev. The minimum is rather flat and there is very little change in the value of absorption co-efficient from energy 5 mc^2 up to 10 mc^2 . The theoretical values of absorption co-efficient have been experimentally verified by several workers, e.g., Meitner and Hupfeld (1930), Chao (1930), Tarrant (1930) and others. But though all of them used adequate precaution to eliminate the effect of secondary radiations they did not use sufficient filter thickness to cut out the softer components. Only 6 or 7 cm. lead was used as the filter thickness. The presence of softer component at this thickness would appreciably increase the absorption co-efficient.

Recently Cork and Pidd (1944) measured the absorption co-efficient of gamma-rays in different materials and for radio-active sodium gamma-rays of energy about 2.8 Mev they found a value much lower than the theoretical value in lead and copper up to about 10 cm. thickness. They concluded that Klein-Nishina formulae for Compton scattering are not valid. Cork (1945) confirmed his result in a subsequent paper. We came across this paper when we had finished our experiment and our results are also similar to that of Cork only under certain experimental condition. But the conclusions of Cork and Pidd about the inefficiency of Klein-Nishina formulae have been contradicted by Gerhart Groet-Zinger and Lloyd-Smith (1945) who measured the absorption co-efficient of radio-sodium gamma-rays with a twofold coincident counter arrangements. The counter thickness was such that only Compton electron of energy more than 2 Mev can pass through both the counters. They found complete agreement with theory. Their results, therefore, show that the anomaly is due to some secondary softer radiation.

EFFECT OF SECONDARY RADIATIONS

Now let us make a brief analysis to what extent different secondary radiations may effect the experimentally determined absorption co-efficient

depending on the geometry of experimental arrangement. Firstly, the probability of Compton scattering as given by (1) for energetic quanta is maximum in the forward direction and so some of the photons, which are presumed to be lost by scattering, may still pass through the measuring instrument and thus reduce the absorption co-efficient. The presence of softer secondaries in the original beam itself, however, would increase the absorption co-efficient.

Secondly corresponding to each photon lost by pair formation we get a positron electron pair emitted within a solid angle approximately $\frac{mc^2}{K_0}$ where K_0 is the energy of the photon. The positron, however, gains in energy by capturing an electron before annihilation. As a matter of fact a slow positron has a negative energy absorption co-efficient and therefore its absorption at low energy will be quite different from that of an electron. The positron may annihilate emitting two photons in the backward and forward directions or a single photon in the forward direction. Dirac (1930) calculated the probability of two quanta annihilation of a positron and showed that it is maximum when the positron is at rest. In general, however, the positron annihilation radiations will have a continuous energy distribution. Experimentally also the disappearance of fast positron in Wilson chamber photographs has been observed by some workers. According to Heitler (*loc. cit.*) again the probability of positron annihilation is maximum when its K. E. is about mc^2 . Therefore the total energy of positron electron system plus the K. E. of the positron will be about 1.5 Mev. Since the energy of the quanta emitted in the backward direction is generally of the order mc^2 the quanta emitted in the forward direction will be of energy about 1 Mev. The whole energy may also be emitted as a single quanta in the forward direction but the probability of one quanta annihilation as calculated by Heitler (*loc. cit.*) is only 20% to that of two-quanta annihilation. This two-quanta annihilation was roughly verified by Klemperer (1933), Thibaud (1933), Joliot (1934) and others. The excess scattering of photons of energy mc^2 in the forward direction, as observed by Gray and Tarrant (1932), is an indirect evidence of two quanta annihilation. Similarly the upward radiation produced by cosmic-rays at higher altitude as observed by Koiff and Clarke (1939) by placing a lead block below the counter may be partly due to upward positron annihilation quanta, for whenever a shower is produced in lead, positrons are generated and these by two-quanta annihilation emit corresponding number of photons in the upward direction. The same positron annihilation photon may also produce excess of low energy Compton electron associated with a shower.

Now the extent to which these positron annihilation radiations can effect the absorption co-efficient, provided the measuring instrument is within the solid angle of pair emission, will depend on what fraction of these can come out of the absorber without scattering or re-absorption. The probability of

absorption for one Mev photon is rather high. But recently Ruark (1945) and others have suggested that when a positron captures an electron the annihilation of the positron electron system is not instantaneous. He also refers to a paper by Wheeler (1946). The name Electro-meson itself suggests that the system can penetrate a large thickness of matter as a cosmic-ray meson before annihilation. If this idea is theoretically sound it is of particular significance in penetrating cosmic-ray cascades and in the absorption of hard gamma-rays capable of generating pair. The effect will be to reduce the absorption co-efficient. Dr. Bhabha, however, is of opinion that such a system can only form an unstable atom and if it is of sufficient K. E. the process will be immediately ionised. But there it will again capture an electron and it will continue till the annihilation.

Thirdly if the pair formed be of sufficient energy they can multiply as in cascade process worked out by Bhabha and Heitler (1927) and others. The effect will be to reduce the absorption co-efficient. But in the gamma-ray region if any quanta are produced by radiation loss it will be of low energy. The critical energy for Pb. is about 10 Mev. Moreover cascade-effect is confined to first few cm. of Pb only.

EXPERIMENTAL PROCEDURE AND ARRANGEMENTS

We repeated Russel's (1913) experiment at higher thickness of lead with gamma-rays from a radon capillary in equilibrium with RaB, RaC, etc. As is well known the penetrating component is only RaC gamma-rays and the rest are cut out at much lower thickness of absorber. Russel measured the absorption co-efficient in mercury up to about 20 cm. for gamma-rays from a 300 mc., radon capillary in equilibrium. A large electroscope was used for the measurement and he found the same absorption co-efficient from about 3 cm., up to about 20 cm. of mercury. From the mass absorption co-efficient of mercury he also deduced the absorption co-efficient for lead to be about $.5 \text{ cm.}^{-1}$. But afterwards his homogeneous absorption co-efficient up to 20 cm. of mercury and lead was doubted by Rutherford, Chadwick, Ellis (*loc. cit.*) and others due to the complexity of gamma-ray spectra revealed by the study of recoil electrons. As stated above the high degree of heterogeneity in the RaC gamma-ray itself has been recently confirmed by Latyshev (*loc. cit.*) and others. Further due to the large dimension of the electroscope and experimental arrangements the secondary effect was too large. Moreover the electroscope he used was of such sensitivity that a fraction less than 2×10^{-6} of the total intensity could not be detected. Now for 300 mc. of radon there will be about 10^{10} disintegrations per second and consequently this much sensitivity is rather too small. We therefore repeated Russel's experiment with a much more sensitive Geiger-Muller counter placing it at different

distances from the absorber surface in order to investigate the nature of secondary effect.

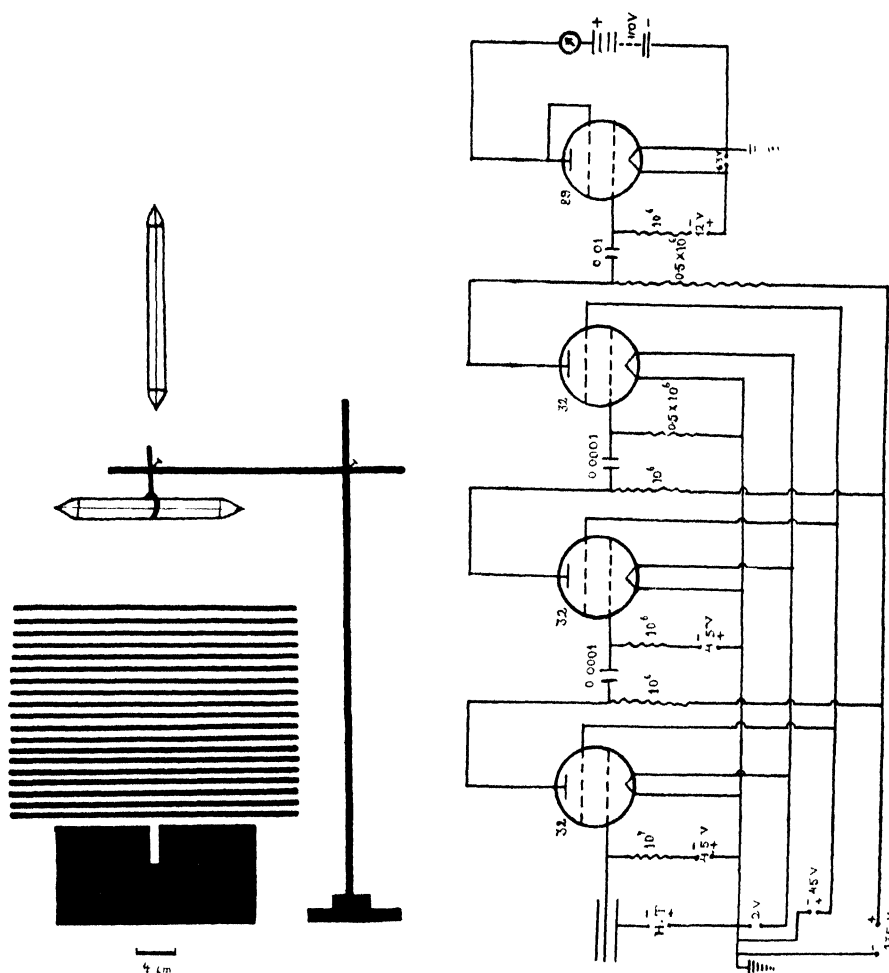


FIG. 1

The experimental arrangement is shown in Fig. 1. A small radon capillary of length about 5 mm. was placed vertically in a hole of diameter about 5 mm. and depth 4 cm., drilled at the centre of a lead block of diameter 21 cm. and thickness 10 cm. Then successive numbers of lead plates of dimension $12'' \times 12'' \times 1/8''$ were placed on the hole symmetrically up to a height about 20 cm. The object of the hole is to give a canalising effect and eliminate as far as possible the effect of an extended source. The corresponding intensity under each thickness of absorber was studied with two Geiger-Muller counters placed horizontally and vertically above the absorber and at different

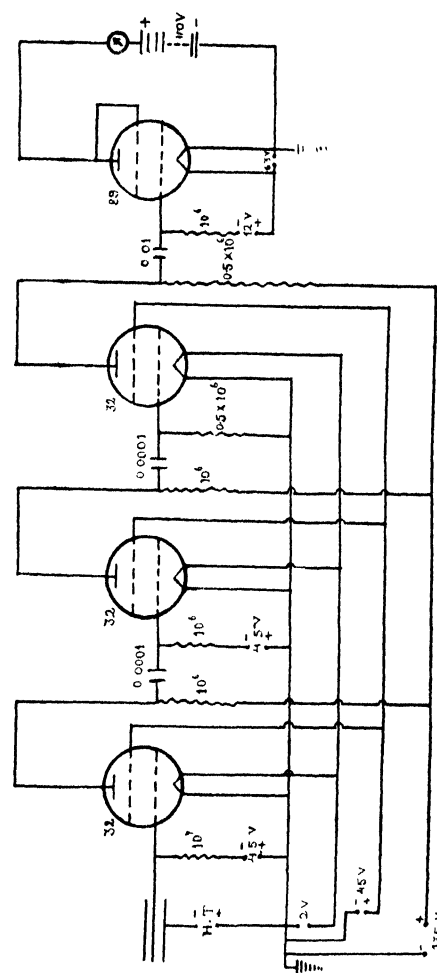


FIG. 2

TABLE I
Distance of the counter above the source = 30 cm.
Counter voltage = 1500 volts.

Amount of Radon	Thickness of Pb. absorber in cm.	No. of count per 2mt. ↓	No. of count per 2mt. ↑	Average No of count per 2 mt.	Absorption co-efficient	Range
56 mc.	21.77	208	197	202.5		
	20.33	297	278	287.5		
	20.01	280	293	286.5		
	19.69	301	305	303.0		
	19.37	312	333	322.5	.18 cm ⁻¹	19.7 & 18.7 cm
	19.06	312	317	314.5		
	18.71	388	375	381.5		
	18.42	416	390	403.0	.52 "	19.1 & 17.8 cm
	18.10	410	135	137.5		
	17.79	180	175	177.5		
	17.47	520	515	517.5		
	17.15	584	567	575.5	.13 "	17.8 & 16.5 cm
	16.83	622	617	619.5		
	16.52	656	695	675.5		
	16.20	768	817	792.5	.10 "	17.2 & 15.9 cm
	15.88	855	835	845.0		

TABLE II
Distance of the Counter above the source = 40 cm.
Counter voltage = 1500 volts.

Amount of Radon	Thickness of Pb. absorber in cm.	Average No of count per 2 mt	Absorption co-efficient	Range
110 mc.	21.77	201		
	20.33	326		
	20.01	351		
	19.69	362	.43 cm. ⁻¹	20.33 & 19.10 cm.
	19.37	394		
	19.06	414	.40 cm. ⁻¹	19.7 & 18.7 cm.
	18.74	436		
	18.42	490	.47 cm. ⁻¹	19.10 & 17.80 cm.
	18.10	521		
	17.79	586		
	17.47	647	.49 cm. ⁻¹	17.47 & 17.15 cm.
	17.15	725		

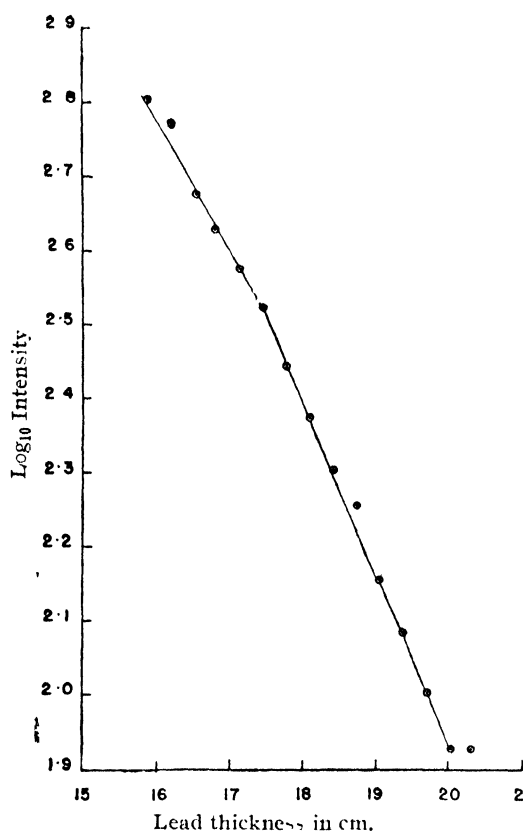


FIG. 3

capable of counting at the rate of about 10 per sec. and the resolving time was much less than what is necessary for the high rate of counting used in these experiments. Data of Table I and Table II are plotted in logarithmic scale against the absorber thickness in Fig. 3 and Fig. 4 respectively.

TABLE III

Counter No. 2 held vertically.

Distance of the bottom of the counter above the source—45 cm.

Pb filter thickness—19 cm.

Amount of Radon	Thickness of Pb. absorber in cm.	No. of counts per units										Absorption Coefficient
		↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	
150 mc.	27.31	300	285		295		290				288	416 ± 0.28 cm^{-1}
	24.77	307	300	321	305	323	296					
	20.33	399	410	435	395	421	405	433	408	426		
	19.06	510		500		510		518	500			

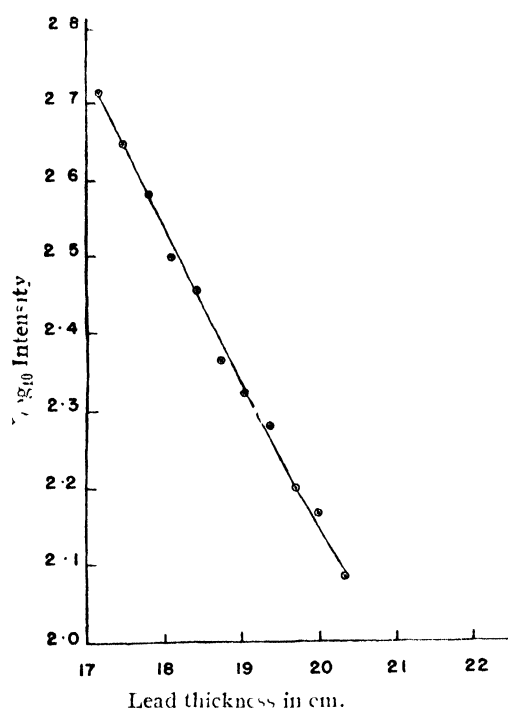


FIG. 4

RESULTS AND DISCUSSION

Table I shows that starting from about 16 cm. thickness of absorber the apparent absorption co-efficient is at first less than the theoretical minimum absorption co-efficient in lead and then gradually increases with the increasing thickness of absorber up to 19.1 cm. The absorption co-efficient between 19.7 and 18.7 cm. is again slightly less and may be due to fluctuation. In order to visualise the effect of fluctuation we repeated the experiment more than once and finally observed the rate of counting when increasing the absorber and then again when decreasing the absorber with a large number of thin lead sheets. From the data it appears that although some error is unavoidable at least the qualitative nature of the result is correct. The logarithmic plot of the data in Fig. 3 also shows that at least two, if not three, different st. lines can be drawn. In this position the counter was 30 cm. above the cylinder surface and due to slight curvature of the lead sheets when the absorber thickness was 20.33 cm., the counter was at a distance about 5 cm. above the absorbing surface and in this position the absorption co-efficient is practically the same as that obtained by Russel. As a matter of fact Russel's experimental condition was such that the absorber itself formed the bottom of the electroscope.

When the counter is 40 cm. above the cylinder surface Table II shows that allowing for some unavoidable fluctuation, the absorption co-efficient steadily decreases with increasing thickness of the absorber as is expected from gradual elimination of the remaining trace of softer radiations. The logarithmic plot

of the data in Fig. 4 also shows a distinct change in slope towards higher thickness. An approximate calculation from the relative intensity distribution of RaC gamma rays as given by Aston and Ellis (*loc. cit.*) and the theoretical absorption co-efficient shows that even after 16 cm. thickness of absorber about 10% of 1.8 Mev gamma-rays remain. The absorption co-efficient of the fraction of gamma-rays between 18.7 to 20.33 cm. of lead fluctuates between $.40 \text{ cm.}^{-1}$ and $.43 \text{ cm.}^{-1}$.

It is therefore clear from the absorption co-efficient measurement in these two positions that as a combined effect of heterogeneity of photon beam and secondary radiation there are two opposite effects on the absorption co-efficient obtained by gradually increasing the absorber thickness. One predominates over the other depending on the geometry of experimental arrangement. When the counter is nearer to the absorbing surface the absorption co-efficient increases with increasing thickness of absorber. As referred by Rutherford, Chadwick, Ellis (*loc. cit.*) similar evidence was obtained by Oba and Bastings. When the counter is further away the absorption co-efficient decreases with increasing thickness. This may be the reason why Russel obtained the same absorption co-efficient from 3 cm. up to 20 cm. of mercury, although RaC gamma-rays are highly heterogeneous. Similarly in other experiments the apparent agreement with theory may be purely due to the balancing of these two opposing effects or due to incomplete filtering.

Further it may be pointed out that in large cosmic-ray showers and bursts we get similar heterogeneous beams of photons and as various geometry of counter arrangements are used to study the shape of transition curves and other cosmic-ray phenomena this experiment may have some indirect significance in those experiments also.

Now as the Table 2 shows that the average absorption co-efficient for gamma-rays filtered through about 18 cm. of lead is $.415 \text{ cm.}^{-1}$, which is more than 10% less than the theoretical minimum absorption co-efficient in lead, we therefore more thoroughly studied the absorption of gamma-rays filtered through 19 cm. of lead with a more sensitive and stable and smaller counter placed vertically at a distance 45 cm. above the source. The number of counts per two minutes was observed several times at random extending over two hours under 19.06, 20.33, 24.77 and 27.31 cm. of lead absorber. The observations at the two latter thicknesses were to notice if there were any fluctuations in the back-ground immediately before and after the measurement under the two former thicknesses. The experimental data are represented in Table III along with the calculated absorption co-efficient and the standard deviation. The absorption co-efficient is $.416 \pm .028 \text{ cm.}^{-1}$ which is the same as the average absorption co-efficient obtained in the previous experiment by placing the counter horizontally at nearly the same distance away from the source. This shows that the lower value is not due to scattered photons as in that case there would have been appreciable difference in absorption co-efficient measured at two different orientations of the counter. As stated above Cork and Pidd

(*loc. cit.*) obtained a value of absorption co-efficient $.405 \text{ cm.}^{-1}$ for 2.8 Mev gamma-rays which is practically the same as that obtained by us for 2.4 Mev gamma-rays of RaC. Theoretically the minimum absorption co-efficient changes very slowly with energy, and therefore there is very little difference in value for this much difference of energy between 2.8 and 2.4 Mev. As we, and particularly Cork and Pidd, have used a highly canalised beam there can be little error due to scattering from an extended source. Cork and Pidd further state that after certain filter thickness an equilibrium is reached between the scattered and the unscattered radiations and the scattered radiations are present in the same ratio both in the incident and in the emergent beam and therefore they cannot affect the value of absorption co-efficient. They do not give any theoretical proof of their statement but our experiment, with the counter in vertical and horizontal position, and their own experiment at lower energy gamma-rays support this statement. With the same geometry of arrangements Cork and Pidd found almost complete agreement or very little difference with the theoretical value for 1.14 and 1.30 Mev gamma-rays, although these photons have also maximum probability of being scattered in the forward direction as that of 2.4 or 2.8 Mev photons. On the other hand their assumption that Klein-Nishina formulae is insufficient, is found to be untenable by the experiment of Gerbert-Groetzingel and Lloyed-Smith (*loc. cit.*), who verified the Klein-Nishina formulae by confining their measurement only to high energy photons. From all these as well as from the fact that the disagreement with theory becomes appreciable only for higher energy photons when the pair formation begins it can be reasonably concluded that this anomaly is due to pair-formation. From the total absorption co-efficient curves for gamma-rays in lead, as plotted by Heitler (*loc. cit.*), it appears that for 2.4 Mev gamma-rays about $1/5$ th of it is due to pair-formation and $1/5$ th due to Compton scattering and a negligible fraction due to photoelectric absorption. When a pair is formed both the positron and the electron will have only 1.2 Mev energy and as such they cannot come out of the absorber nor can they emit any radiation of appreciable energy as in a cascade. But since a positron gains in energy by capturing an electron before annihilation therefore positron annihilation radiation is likely to be the main cause of disagreement with the theory. Cork further states that in copper the pair formation is negligible and as the disagreement exists there also it is not due to pair formation. But if the value of absorption co-efficient calculated by them for pair formation in copper is subtracted from the total theoretical absorption co-efficient the experimental value is brought much nearer to the theoretical value and moreover, if Wheeler's idea of electro-meson is true, then the annihilation radiation from the filter may also affect the result. Of course, as kindly pointed out by Professor S. N. Bose, at higher energy the probability of multiple scattering will increase but just as scattering cannot effect the absorption co-efficient measurement similarly multiple scattering which is of still higher order may

not affect the absorption co-efficient appreciably. We hope to further investigate this point in future.

Another interesting fact, observed in the last experiment, was that when the radon source was placed in the cavity the background rate of counting, which practically remained steady after 24.77 cm. of lead absorber, was nearly double to that due to cosmic-rays alone when the source was not there. As shown from Table III the background count per 2 mt. is nearly 300, whereas the average background rate of counting, observed for about 15 minutes immediately before placing the source and just after its removal, is 157 ± 8 . We carefully searched for any contamination in lead sheets, radon carrier, etc., but this was completely absent. We did not notice this in the two previous experiments and we took the rate of counting after 24.77 cm. of lead as the natural background. This might be partly due to the fact that we were not so careful and partly due to the fact that comparatively smaller amount of radon was used in those experiments. The difference of the last experimental conditions from the two previous ones was that (1) the counter was held vertically, (2) a strong radon source was used and (3) the source was enclosed by another platino-iridium tube. Although we are not sure of its chemical composition, it is as tested by a magnet not a very light element so that neutron may be emitted by photo disintegration of the nucleus. Moreover the counter was an ordinary copper glass counter filled with argon and petroleum-ether and therefore its probability of neutron counting was very small. Therefore this excess of background may be merely due to some multiple scattered photon reaching the counter other than through the absorber or due to some meson type of radiation emitted by the source or the container. Now the minimum thickness of lead that a photon will have to traverse in coming out of the cylinder is about 7 cm. at the bottom and 11 cm. by the side of the cylinder and then it will have to suffer back reflection and multiple scattering in order to reach the counter and as such its probability is very small. We cannot be, however, sure of it unless more lead sheets are used at the bottom and by the sides of the cylinder. As we have exhausted all the lead sheets we could acquire at present it is not possible to further elucidate this point. But the probability of the residual counting being due to some meson-like emission by the source or the container may not be impossible. We hope to investigate this point further in near future. But it should be mentioned that even if this double background is due to some scattered quanta reaching the counter other than through the absorber it cannot effect the absorption co-efficient calculated with this steady background.

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